On the extension-closed subcategories of the Cohen-Macaulay module category over simple singularities

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This talk is based on joint work with Ryo Takahashi. Throughout this talk, let k be an algebraically closed uncountable field of characteristic zero, and let R be a complete local hypersurface over k. We denote by $\mathrm{mod}\,R$ the category of finitely generated R-modules, by $\mathrm{CM}(R)$ the full subcategory of $\mathrm{mod}\,R$ consisting of all maximal Cohen-Macaulay R-modules, by $\underline{\mathrm{CM}}(R)$ the stable category of $\mathrm{CM}(R)$, by $\mathrm{CM}_0(R)$ the full subcategory of $\mathrm{CM}(R)$ consisting of maximal Cohen-Macaulay modules which are locally free on the punctured spectrum of R.

By Takahashi [3, 4], there are one-to-one correspondences between the following:

- the resolving subcategories of $\mathrm{mod}\,R$ contained in $\mathrm{CM}(R)$,

- the thick subcategories of $\underline{\mathrm{CM}}(R)$,

- the specialization-closed subsets of the singular locus of R.

The bijections are explicitly given, which leads complete classifications of the resolving subcategories and thick subcategories mentioned above. Moreover, we assume that R has only an isolated singularity, then both the resolving subcategories of $\mathrm{mod}\,R$ contained in $\mathrm{CM}(R)$ and the thick subcategories of $\underline{\mathrm{CM}}(R)$ are only trivial.

By definition, both resolving subcategory and thick subcategory are extension-closed. The purpose of this talk is to classify all extension-closed subcategories of $\mathrm{CM}(R)$ and $\underline{\mathrm{CM}}(R)$, when R has only an isolated singularity. The main results in this talk is the following theorem.

Theorem 1 [1] If R is a complete local hypersurface over k, then the following hold.

(1) If either R is artinian hypersurface or R has simple singularity and $\dim R = 2$, then extension-closed subcategories of $\mathrm{CM}(R)$ are only trivial.

(2) If R is 1-dimensional simple singularity and domain, then extension-closed subcategories of $\mathrm{CM}(R)$ are only trivial.

(3) If R is not domain and 1-dimensional simple singularity, $\mathrm{CM}(R)$ has non-trivial extension-closed subcategories.

(4) If R is either 1-dimensional (A_∞) type or 2-dimensional (D_∞) type, then extension-closed subcategories of $\mathrm{CM}_0(R)$ are only trivial.

(5) If R is either 1-dimensional (D_∞) type or 2-dimensional (A_∞) type, then $\mathrm{CM}_0(R)$ has non-trivial extension-closed subcategories.

References

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